ECS 315: In-Class Exercise Solution

Instructions

1. Separate into groups of no more than three persons.
2. **The group cannot be the same as your former group.**
3. Only one submission is needed for each group.
4. ***Write down all the steps*** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
5. **Do not panic.**
6. **Only this page will be scanned and graded. Work only on this page.**

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Consider a random variable whose pmf is given by $p_X(x) = \begin{cases} \frac{c}{x^2}, & x = -2, 1, 3, \\ 0, & \text{otherwise.} \end{cases}$.

a) Check that $c = \frac{36}{49}$.

$$\sum_x p_X(x) = p_X(-2) + p_X(1) + p_X(3) = \frac{36}{49}\times\frac{1}{4} + \frac{36}{49}\times 1 + \frac{36}{49}\times\frac{1}{9} = \frac{36}{49}\left(\frac{9}{36} + \frac{36}{36} + \frac{4}{36}\right)$$
$$= \frac{36}{49}\times\frac{49}{36} = 1 \checkmark$$

b) Find $\mathbb{E}X$

$$\mathbb{E}X = \sum_x x\, p_X(x) = \left((-2)\times\frac{1}{4} + (1)\times 1 + (3)\times\frac{1}{9}\right)\times\frac{36}{49} = \left(-\frac{1}{2} + 1 + \frac{1}{3}\right)\times\frac{36}{49}$$
$$= \frac{5}{6}\times\frac{36}{49} = \frac{30}{49} \approx 0.6122$$

c) Let $Y = (X-2)^2$. $Y = g(X)$ where $g(x) = (x-2)^2$

a. Find $p_Y(y)$.

$p_X(x)$	x	$y = (x-2)^2$
$c/4$	-2	$4^2 = 16$
$c/1$	1	$(-1)^2 = 1$
$c/9$	3	$1^2 = 1$

$S_Y = \{1, 16\}$

$P_Y(1) = P_X(1) + P_X(3)$
$= \left(1 + \frac{1}{9}\right)\times\frac{36}{49} = \frac{40}{49}$

$P_Y(16) = P_X(-2) = \frac{1}{4}\times\frac{36}{49} = \frac{9}{49}$

$$P_Y(y) = \begin{cases} 40/49 \approx 0.8163, & y = 1, \\ 9/49 \approx 0.0204, & y = 16, \\ 0, & \text{otherwise.} \end{cases}$$

b. Find $\mathbb{E}Y$.

$$\mathbb{E}Y = \sum_y y\, P_Y(y) = 1\times\frac{40}{49} + 16\times\frac{9}{49} = \frac{184}{49} \approx 3.7551$$

$$\mathbb{E}Y = \mathbb{E}\left[(X-2)^2\right] = \mathbb{E}\left[X^2 - 4X + 4\right] = \mathbb{E}[X^2] - 4\mathbb{E}X + 4.$$

To find $\mathbb{E}[X^2]$. Let $Z = X^2$

$p_X(x)$	x	$z = x^2$
$c/4$	-2	4
$c/1$	1	1
$c/9$	3	9

So,

$$p_Z(z) = \begin{cases} c/4, & z = 4, \\ c, & z = 1, \\ c/9, & z = 9. \end{cases}$$

$$\mathbb{E}Z = \frac{c}{4} \times 4 + c \times 1 + \frac{c}{9} \times 9 = 3c$$

Alternatively, from LOTUS,

$$\mathbb{E}[X^2] = \sum_x x^2 p_X(x) = \sum_x x^2 \times \frac{c}{x^2} = 3c$$

same

Therefore, $\mathbb{E}Y = 3 \times \frac{36}{49} - 4 \times \frac{30}{49} + 4 \approx 3.7551$ (same as above)